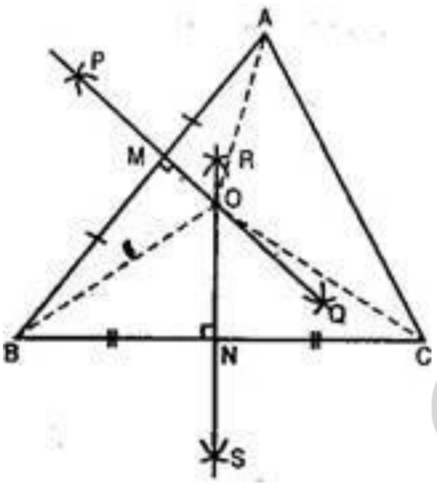
Exercise-7.5

1. ABC is a triangle. Locate a point in the interior of $\triangle$ABC which is equidistant from all the vertices of $\triangle$ABC.

Ans. The point which is equidistant from all the vertices of a triangle is known as the circumcentre of the triangle. This point acts as the centre of a circle which can be drawn by passing through the vertices of the given triangle. And to find out the circumcentre we usually, draw the perpendicular bisectors of any two sides, their point of intersecton is the required point which is equdistant from the vertices (being the radius). So we will proceed with drawing a circumcentre.

Let ABC be a triangle.



Draw perpendicular bisectors PQ and RS of sides AB and BC respectively of triangle ABC. Let PQ bisects AB at M and RS bisects BC at point N.

Let PQ and RS intersect at point O.

Join OA, OB and OC.

Now in $\triangle$AOM and $\triangle$ BOM,

AM = MB [By construction]

$\angle$ AMO = $\angle$ BMO = 90° [By construction]

OM = OM [Common]

$\therefore$ ΔAOM $\cong$ ΔBOM [By SAS congruency]

$\Rightarrow$ OA = OB [By C.P.C.T.] .....(i)

Similarly, ΔBON $\cong$ ΔCON $\Rightarrow$

OB = OC [By C.P.C.T.] .....(ii)

From eq. (i) and (ii),

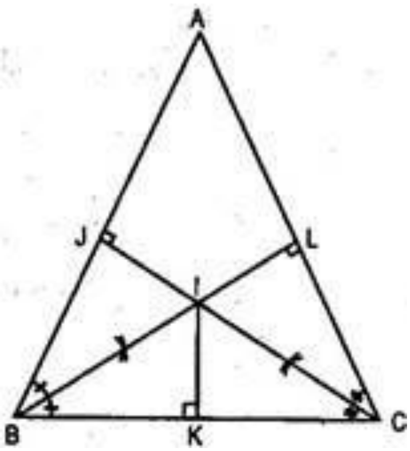
OA = OB = OC

Hence O, the point of intersection of perpendicular bisectors of any two sides of ΔABC equidistant from its vertices.

2. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Ans. The point which is equidistant from all the sides of a triangle is known as its incentre and is the point of intersection of the angle bisectors. Hence we will proceed with finding the incentre of the given triangle.

Let ABC be a triangle.



Draw bisectors of $\angle$ B and $\angle$ C.

Let these angle bisectors intersect each other at point I.

Draw IK $\perp$ BC

Also draw IJ $\perp$ AB and IL $\perp$ AC.

Join AI.

In $\triangle$BIK and $\triangle$BIJ,

$\angle$IKB = $\angle$IJB = 90° [By construction]

$\angle$IBK = $\angle$IBJ

[$\because$ BI is the bisector of $\angle$B (By construction)]

BI = BI [Common]

$\therefore$ $\triangle$BIK $\cong$ $\triangle$BIJ [ASA criteria of congruency]

$\therefore$ IK = IJ [By C.P.C.T.] .... (i)

Similarly, $\triangle$CIK $\cong$ $\triangle$CIL

$\therefore$ IK = IL [By C.P.C.T.] .... (ii)

From eq (i) and (ii),

IK = IJ = IL

Hence, I is the point of intersection of angle bisectors of any two angles of $\triangle$ABC equidistant from its sides.

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3. In a huge park, people are concentrated at three points (See figure).

A: where there are different slides and swings for children.

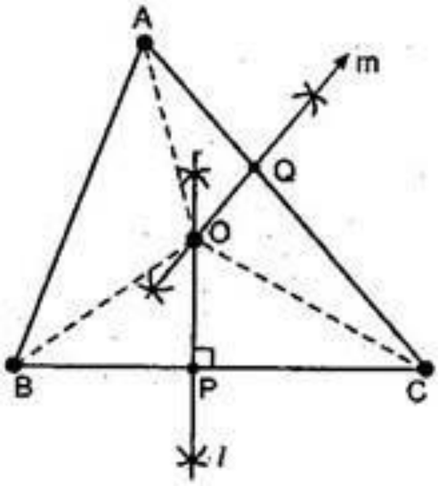
B: near which a man-made lake is situated.

C: which is near to a large parking and exit.

Where should an ice cream parlour be set up so that maximum number of persons can approach it?

Ans. The parlour should be equidistant from A, B and C. So we should find out the circumcentre of the triangle obtained by joining A, B and C respectively.

For this let us draw perpendicular bisector say l of line joining points B and C also draw perpendicular bisector say m of line joining points A and C.



Let l and m intersect each other at point O. O is the required point.

Proof that O is the required point:

Join OA, OB and OC.

Proof: In ΔBOP and ΔCOP,

OP = OP [Common]

$\angle$OPB = $\angle$OPC = 90°

BP = PC [P is the mid-point of BC]

$\therefore$ ΔBOP $\cong$ ΔCOP [By SAS congruency]

$\Rightarrow$ OB = OC [By C.P.C.T.] .....(i)

Similarly, ΔAOQ $\cong$ ΔCOQ

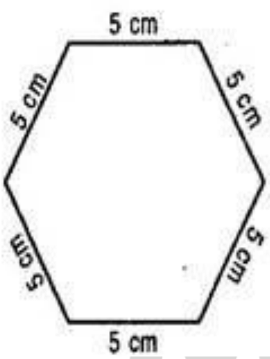
$\Rightarrow$ OA = OC [By C.P.C.T.] .....(ii)

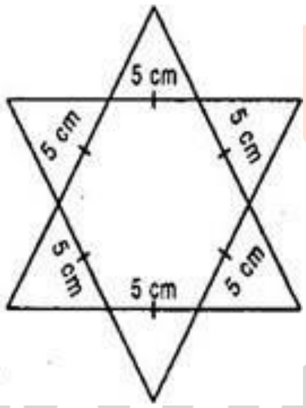
From eq. (i) and (ii),

OA = OB = OC

Therefore, O is the required point as it is equidistant from the given points. Thus, ice cream parlour should be set up at point O, the point of intersection of perpendicular bisectors of any two sides out of three formed by joining these points.

4. Complete the hexagonal rangoli and the star rangolies (See figure) but filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?





Ans. In hexagonal rangoli, Number of equilateral triangles each of side 5 cm are 6.

Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(5)^2 = \frac{\sqrt{3}}{4}\times 25$ sq. cm

Area of hexagonal rangoli = 6 x Area of an equilateral triangle

= $6\times\frac{\sqrt{3}}{4}\times 25 = 150\times\frac{\sqrt{3}}{4}$ sq. cm...............(i)

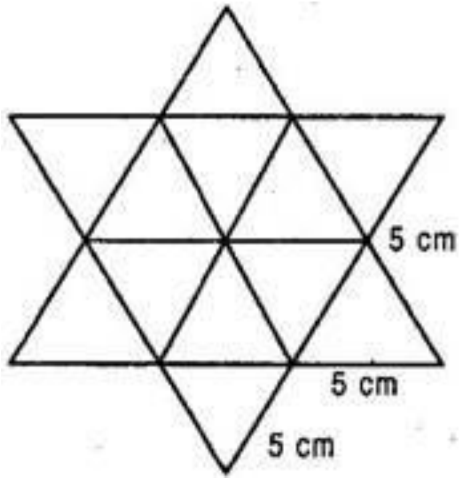
Now area of equilateral triangle of side 1 cm = = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$ sq. cm....... (ii)

Number of equilateral triangles each of side 1 cm in hexagonal rangoli

$= 150 \times \frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4} = 150 \times \frac{\sqrt{3}}{4} \times \frac{4}{\sqrt{3}}$ = 150 .....(iii)

Now in Star rangoli,

Number of equilateral triangles each of side 5 cm = 12



Therefore, total area of star rangoli = 12 × Area of an equilateral triangle of side 5 cm

$= 12 \times \left(\frac{\sqrt{3}}{4} (5)^2 \right)$

$= 12 \times \frac{\sqrt{3}}{4} \times 25$

$= 300 \frac{\sqrt{3}}{4}$ sq. cm..............(iv)

Number of equilateral triangles each of side 1 cm in star rangoli

$= 300 \frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4}$

$= 300 \frac{\sqrt{3}}{4} \times \frac{4}{\sqrt{3}}$

= 300 .... (v)

From eq. (iii) and (v), we observe that star rangoli has more equilateral triangles each of side 1 cm.