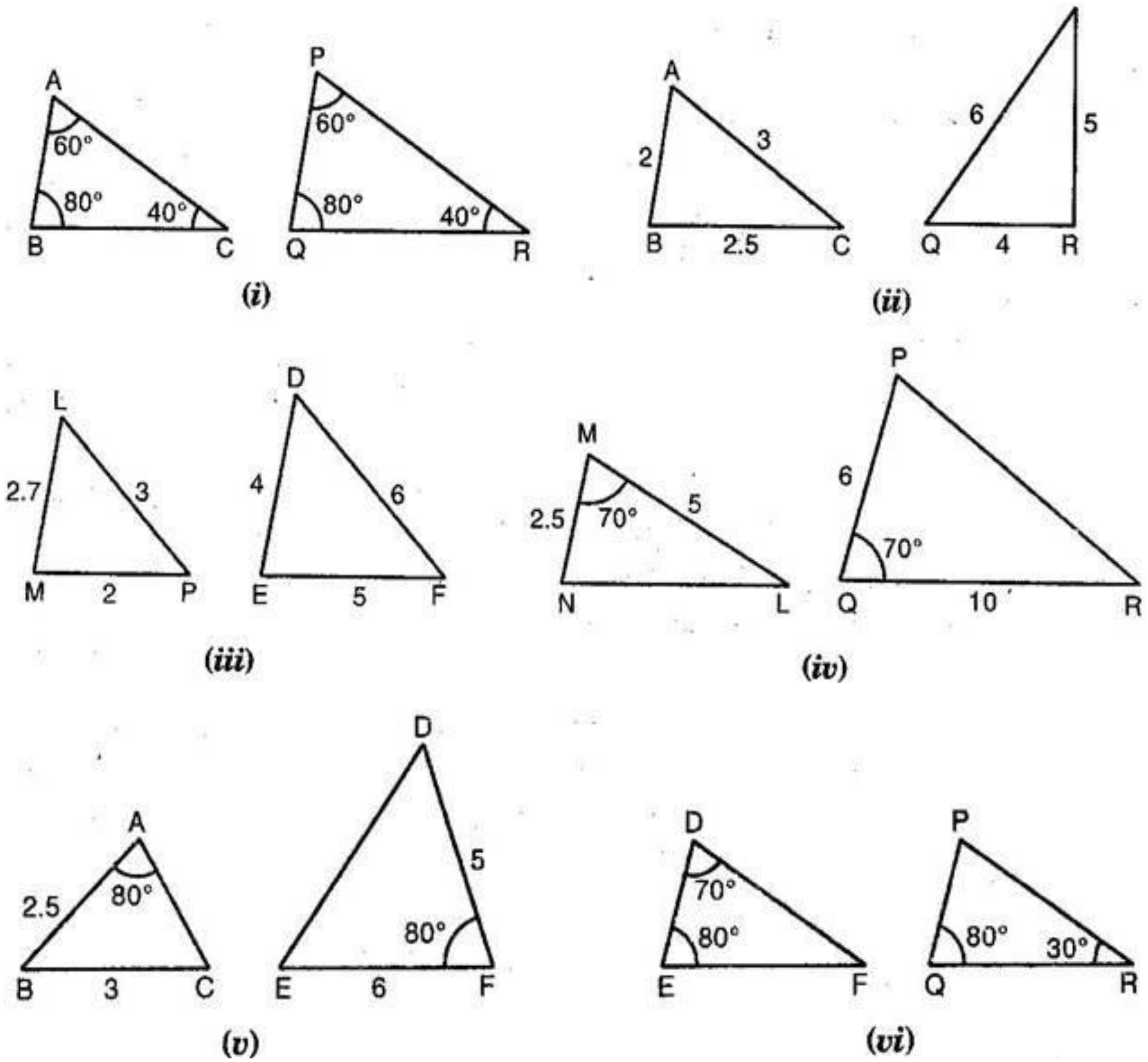
Exercise 6.3

1. State which pairs of triangles in the given figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Ans. (i) In Δs ABC and PQR, we observe that,

$\angle A = \angle P = 60^\circ$, $\angle B = \angle Q = 80^\circ$ and $\angle C = \angle R = 40^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta ABC \sim \Delta PQR$

(ii) In Δs ABC and PQR, we observe that,

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ} = \frac{1}{2}$$

$\therefore$ By SSS criterion of similarity, $\Delta ABC \sim \Delta PQR$

(iii) In Δs LMP and DEF, we observe that the ratio of the sides of these triangles is not equal.

Therefore, these two triangles are not similar.

(iv) In Δs MNL and QPR, we observe that, $\angle M = \angle Q = 70^\circ$

But, $\frac{MN}{PQ} \neq \frac{ML}{QR}$

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

(v) In Δs ABC and FDE, we have, $\angle A = \angle F = 80^\circ$

But, $\frac{AB}{DE} \neq \frac{AC}{DF}$ [$\because$ AC is not given]

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

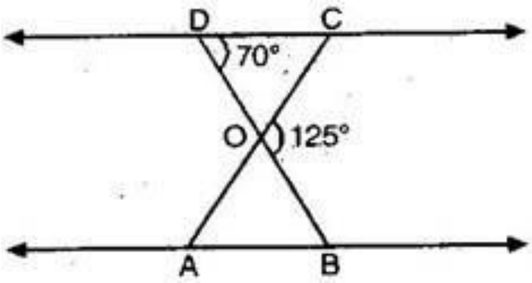
(vi) In Δs DEF and PQR, we have, $\angle D = \angle P = 70^\circ$

[$\because$] $\angle P = 180^\circ - 80^\circ - 30^\circ = 70^\circ$

And $\angle E = \angle Q = 80^\circ$

$\therefore$ By AAA criterion of similarity $\Delta DEF \sim \Delta PQR$

2. In figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Ans. Since BD is a line and OC is a ray on it.

$\therefore \angle DOC + \angle BOC = 180^\circ$

$\Rightarrow \angle DOC + 125^\circ = 180^\circ$

$\Rightarrow \angle DOC = 55^\circ$

In $\triangle$ CDO, we have $\angle CDO + \angle DOC + \angle DCO = 180^\circ$

$\Rightarrow 70^\circ + 55^\circ + \angle DCO = 180^\circ$

$\Rightarrow \angle DCO = 55^\circ$

It is given that $\triangle ODC \sim \triangle OBA$

$\therefore \angle OBA = \angle ODC, \angle OAB = \angle OCD$

$\Rightarrow \angle OBA = 70^\circ, \angle OAB = 55^\circ$

Hence $\angle DOC = 55^\circ$, $\angle DCO = 55^\circ$ and $\angle OAB = 55^\circ$

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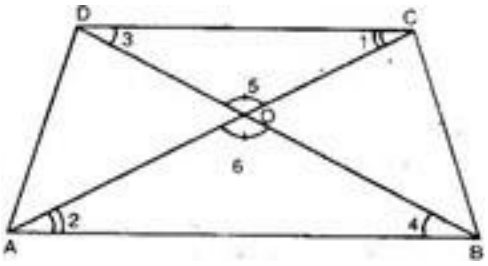
3. Diagonals AC and BD of a trapezium ABCD with AB || CD intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Ans. Given: ABCD is a trapezium in which AB $\parallel$ DC.



To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In Δs OAB and OCD, we have,

$\angle 5 = \angle 6$ [Vertically opposite angles]

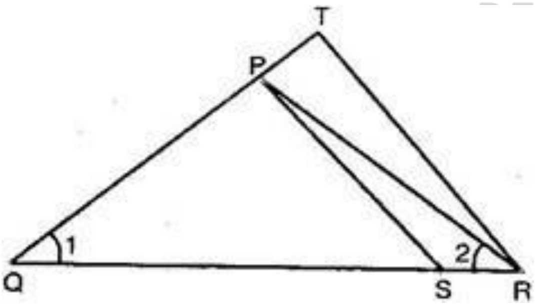
$\angle 1 = \angle 2$ [Alternate angles]

And $\angle 3 = \angle 4$ [Alternate angles]

$\therefore$ By AAA criterion of similarity, $\Delta OAB \sim \Delta ODC$

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Ans. We have, $\frac{QR}{QS} = \frac{QT}{PR}$

$\Rightarrow \frac{QT}{QR} = \frac{PR}{QS}$ ....(1)

Also, $\angle 1 = \angle 2$ [Given]

$\therefore$ PR = PQ ....(2) [$\because$ Sides opposite to equal $\angle$ s are equal]

From eq.(1) and (2), we get

$$\frac{QT}{QR} = \frac{PR}{QS} \Rightarrow \frac{PQ}{QT} = \frac{QS}{QR}$$

In Δs PQS and TQR, we have,

$\frac{PQ}{QT} = \frac{QS}{QR}$ and $\angle$ PQS = $\angle$ TQR = $\angle$ Q

$\therefore$ By SAS criterion of similarity, Δ PQS ~ Δ TQR

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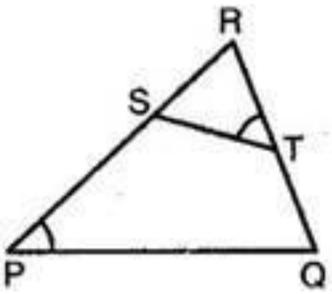
5. S and T are points on sides PR and QR of a Δ PQR such that $\angle$ P = $\angle$ RTS. Show that Δ RPQ ~ Δ RTS.

Ans. In Δs RPQ and RTS, we have



$\angle$ RPQ = $\angle$ RTS [Given]

$\angle$ PRQ = $\angle$ TRS [Common]

$\therefore$ By AA-criterion of similarity,

Δ RPQ ~ Δ RTS

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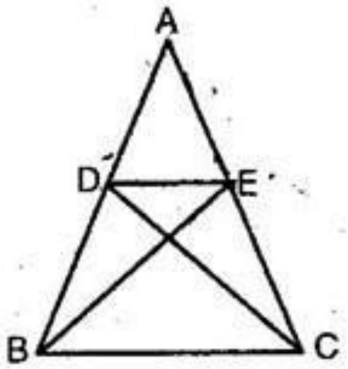
6. In the given figure, if Δ ABE $\cong$ Δ ACD, show that Δ ADE ~ Δ ABC.



Ans. It is given that $\triangle ABE \cong \triangle ACD$

$\therefore$ AB = AC and AE = AD

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE} \quad \text{..........(1)}$$

$\therefore$ In $\triangle$s ADE and ABC, we have,

$$\frac{AB}{AC} = \frac{AD}{AE} \text{ [from eq.(1)]}$$

And $\angle BAC = \angle DAE$ [Common]

Thus, by SAS criterion of similarity, $\triangle ADE \sim \triangle ABC$

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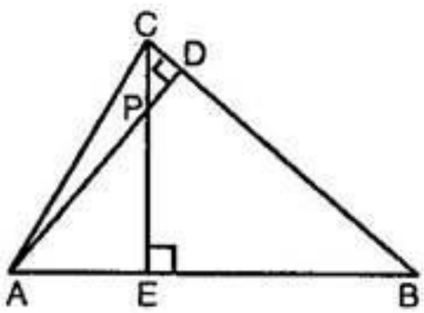
7. In figure, altitude AD and CE of a $\triangle$ABC intersect each other at the point P. Show that:



(i) $\triangle$AEP $\sim$ $\triangle$CDP

(ii) Δ **ABD** $\sim$ Δ **CBE**

(iii) Δ **AEP** $\sim$ Δ **ADB**

(iv) Δ **PDC** $\sim$ Δ **BEC**

Ans. (i) In Δs AEP and CDP, we have,

$\angle AEP = \angle CDP = 90^\circ$ [$\because$ $CE \perp AB, AD \perp BC$]

And $\angle APE = \angle CPD$ [Vertically opposite]

$\therefore$ By AA-criterion of similarity, $\Delta AEP \sim \Delta CDP$

(ii) In Δs ABD and CBE, we have,

$\angle ADB = \angle CEB = 90^\circ$

And $\angle ABD = \angle CBE$ [Common]

$\therefore$ By AA-criterion of similarity, $\Delta ABD \sim \Delta CBE$

(iii) In Δs AEP and ADB, we have,

$\angle AEP = \angle ADB = 90^\circ$ [$\because$ $AD \perp BC, CE \perp AB$]

And $\angle PAE = \angle DAB$ [Common]

$\therefore$ By AA-criterion of similarity, $\Delta AEP \sim \Delta ADB$

(iv) In Δs PDC and BEC, we have,

$\angle PDC = \angle BEC = 90^\circ$ [$\because$ $CE \perp AB, AD \perp BC$]

And $\angle PCD = \angle BEC$ [Common]

$\therefore$ By AA-criterion of similarity, $\Delta PDC \sim \Delta BEC$

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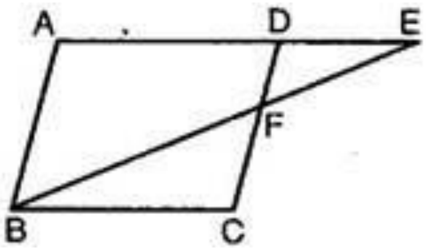
8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE $\sim$ Δ CFB.

Ans. In Δs ABE and CFB, we have,



$\angle$ AEB = $\angle$ CBF[Alt. $\angle$ s]

$\angle$ A = $\angle$ C [opp. $\angle$ s of a $\parallel$ gm]

$\therefore$ By AA-criterion of similarity, we have

ΔABE ~ ΔCFB

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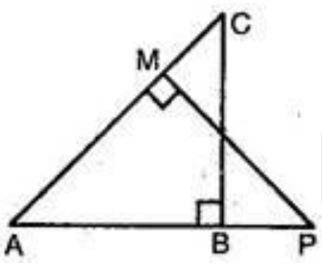
9. In the given figure, ABC and AMP are two right triangles, right angles at B and M respectively. Prove that:



(i) Δ**ABC ~** Δ **AMP**

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Ans. (i) In Δs ABC and AMP, we have,

$\angle$ ABC = $\angle$ AMP = 90° [Given]

$\angle$ BAC = $\angle$ MAP [Common angles]

$\therefore$ By AA-criterion of similarity, we have

ΔABC ~ ΔAMP

(ii) We have $\Delta ABC \sim \Delta AMP$ [As prove above]

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$$

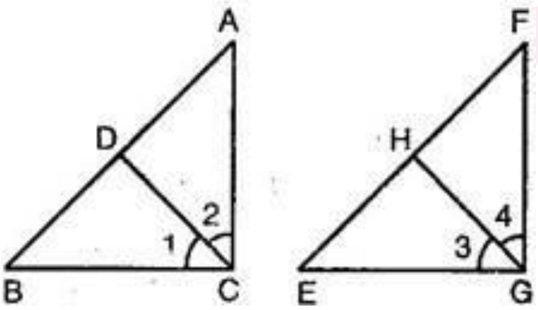
10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE at ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HE$

(iii) $\Delta DCA \sim \Delta HGF$

Ans. We have, $\Delta ABC \sim \Delta FEG$



$\Rightarrow \angle A = \angle F$.........(1)

And $\angle C = \angle G$

$$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$$

$\Rightarrow \angle 1 = \angle 3$ and $\angle 2 = \angle 4$...........(2)

[$\because$ CD and GH are bisectors of $\angle C$ and $\angle G$ respectively]

$\therefore$ In Δs DCA and HGF, we have

$\angle A = \angle F$[From eq.(1)]

$\angle 2 = \angle 4$ [From eq.(2)]

$\therefore$ By AA-criterion of similarity, we have

$\Delta DCA \sim \Delta HGF$

Which proves the (iii) part

We have, $\Delta DCA \sim \Delta HGF$

$$\Rightarrow \frac{AG}{FG} = \frac{CD}{GH}$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

Which proves the (i) part

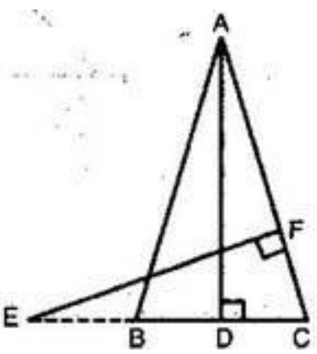
In Δs DCA and HGF, we have

$\angle 1 = \angle 3$ [From eq.(2)]

$\angle B = \angle E$ [$\because \Delta DCB \sim \Delta HE$]

Which proves the (ii) part

11. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that $\Delta ABD \sim \Delta ECF$.



Ans. Here ΔABC is isosceles with AB = AC

$\therefore \angle B = \angle C$

In $\triangle$s ABD and ECF, we have

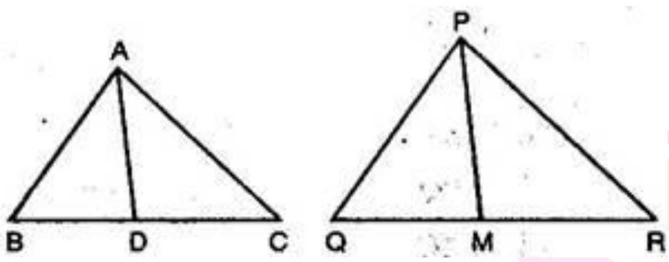
$\angle$ ABD = $\angle$ ECF [$\because$ $\angle$ B = $\angle$ C]

$\angle$ ABD = $\angle$ ECF = 90° [$\because$ AD $\perp$ BC and EF $\perp$ AC]

$\therefore$ By AA-criterion of similarity, we have

$\triangle$ ABD ~ $\triangle$ ECF

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of a $\triangle$PQR (see figure). Show that $\triangle$ABC ~ $\triangle$PQR.



Ans. Given: AD is the median of $\triangle$ ABC and PM is the median of $\triangle$ PQR such that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To prove: $\triangle$ ABC ~ $\triangle$ PQR

Proof: BD = $\frac{1}{2}$ BC [Given]

And QM = $\frac{1}{2}$ QR [Given]

Also $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\therefore$ $\Delta ABD \sim \Delta PQM$[By SSS-criterion of similarity]

$\Rightarrow \angle B = \angle Q$[Similar triangles have corresponding angles equal]

And $\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

$\therefore$ By SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta PQR$

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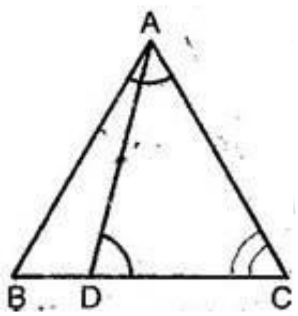
13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

ANS. In triangles ABC and DAC,



$\angle ADC = \angle BAC$ [Given]

and $\angle C = \angle C$[Common]

$\therefore$ By AA-similarity criterion,

$\Delta ABC \sim \Delta DAC$

$$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$

$$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$$

$\Rightarrow CA^2 = CB.CD$

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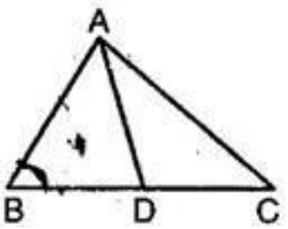
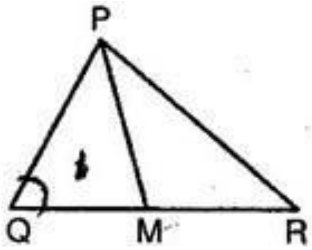
14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle$ABC ~ $\triangle$PQR.

ANS. Given: AD is the median of $\triangle$ ABC and PM is the median of $\triangle$ PQR such that



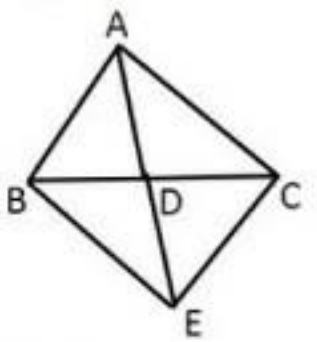


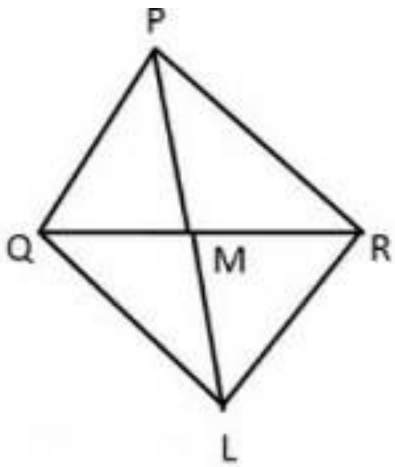
$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \ldots\ldots\ldots(1)$$

To prove: $\triangle$ ABC ~ $\triangle$ PQR

Proof:

Let us extend AD to point D such that AD = DE and PM up to point L such that PM = ML





Join B to E. C to E,and Q to L, and R to L

We know that medians is the bisector of opposite side

Hence

BD = DC

Also, AD = DE (by construction)

Hence in quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

AC = BE

AB = EC (opposite sides of | | gm are equal) ....(2)

Similarly, we can prove that PQLR is a parallelogram

PR = QL

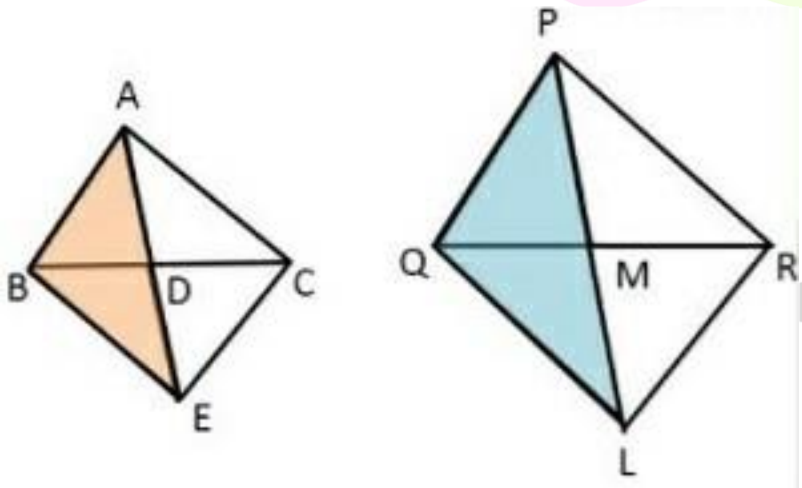
PQ = LR opposite sides of | | gm areequal)............ (3)

Given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$
$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AD}{PM}$$ [from (2) (3)]
$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL} \; [as\; AD = DE, AE = AD + DE = 2AD$$
$$PM = ML.PL = PM + ML = 2PM$$

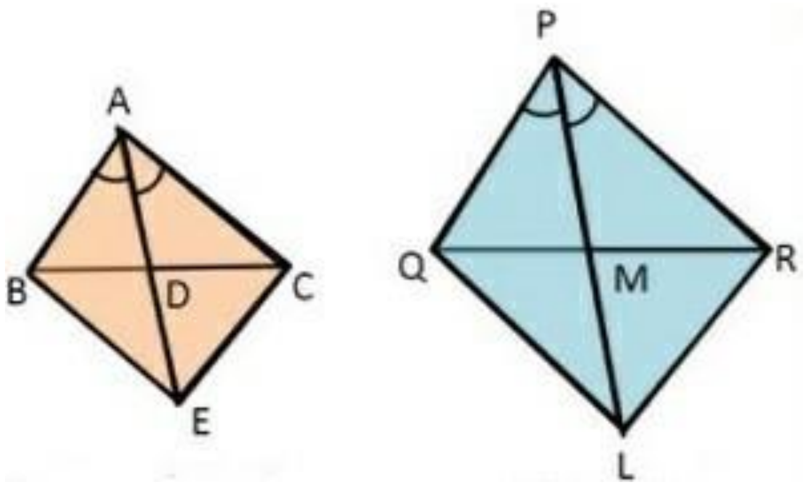
$\triangle ABE \sim \triangle PQL$ (By SSS Similiarity Criteria)



We know that corresponding angles of similar triangles are equal.

$\angle BAE = \angle QPL$ (4)

Similarly, we can prove that $\triangle AEC \sim \triangle PLR$.



We know that corresponding angles of similar triangles are equal.

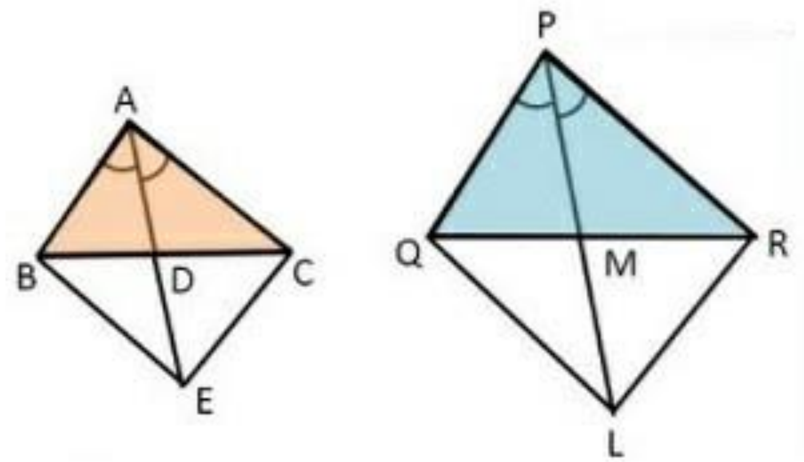
$\angle CAE = \angle RPL$ (5)

Adding (4) and (5),

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\angle CAB = \angle RPQ$

$In \triangle ABC \text{ and } \triangle PQR,$



$\frac{AB}{PQ} = \frac{AC}{PR}$

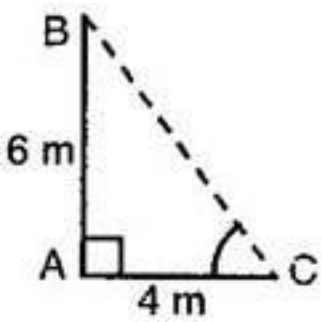
$\angle CAB = \angle RPQ$

$\triangle ABC \sim \triangle PQR$

Hence proved

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Ans. Let AB the vertical pole and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Joined BC and EF.



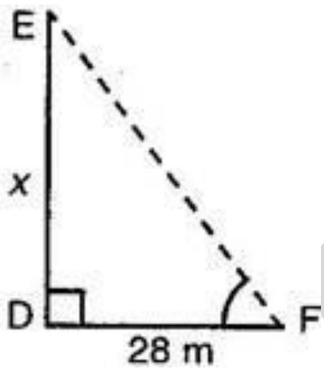
Let DE= x meters

Here, AB = 6 m, AC = 4 m and DF = 28 m

In the triangles ABC and DEF,

$\angle A = \angle D = 90^\circ$

And $\angle C = \angle F$[Each is the angular elevation of the sun]



$\therefore$ By AA-similarity criterion,

$\triangle ABC \sim \triangle DEF$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{6}{x} = \frac{4}{28}$

$\Rightarrow \frac{6}{x} = \frac{1}{7}$

$\Rightarrow x = 42$ m

Hence, the height of the tower is 42 meters.

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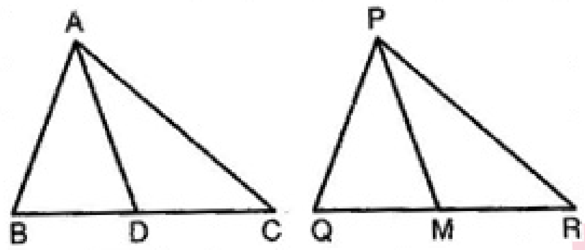
16. If AD and PM are medians of triangles ABC and PQR respectively, where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Ans. Given: AD and PM are the medians of triangles

ABC and PQR respectively, where



$\Delta ABC \sim \Delta PQR$

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof: In triangles ABD and PQM,

$\angle B = \angle Q$ [Given]

And $\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$ [$\because$ AD and PM are the medians of BC and QR respectively]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

$\therefore$ By SAS-criterion of similarity,

$\Delta ABD \sim \Delta PQM$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$