Exercise 3.5

1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0

3x – 9y – 2 = 0

(ii)$2x + y = 5$

$3x + 2y = 8$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Ans. (i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Comparing equation $x - 3y - 3 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 9y - 2 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -3, a_2 = 3, b_2 = -9, c_2 = -2$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ this means that the two lines are parallel.

Therefore, there is no solution for the given equations i.e. it is inconsistent.

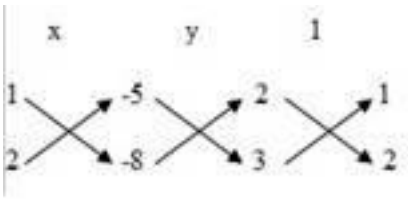
(ii) $2x + y = 5$

$3x + 2y = 8$

Comparing equation $2x + y = 5$ with $a_1x+b_1y+c_1 = 0$ and $3x + 2y = 8$ with $a_2x+b_2y+c_2 = 0$,

We get $a_1 = 2, b_1 = 1, c_1 = -5, a_2 = 3, b_2 = 2, c_2 = -8$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that there is unique solution for the given equations.



$$\frac{x}{(-8)(1)-(2)(-5)} = \frac{y}{(-5)(3)-(-8)(2)} = \frac{1}{(2)2-(3)1}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow x = 2$ and $y = 1$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

Comparing equation $3x - 5y = 20$ with $a_1x+b_1y+c_1 = 0$ and $6x - 10y = 40$ with $a_2x+b_2y+c_2 = 0$,

We get $a_1 = 3, b_1 = -5, c_1 = -20, a_2 = 6, b_2 = -10, c_2 = -40$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

It means lines coincide with each other.

Hence, there are infinitely many solutions.

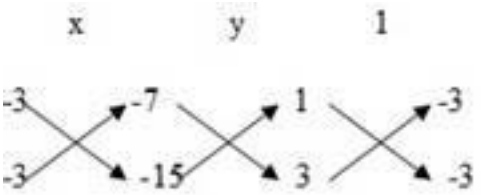
(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Comparing equation $x - 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 3y - 15 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -7, a_2 = 3, b_2 = -3, c_2 = -15$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that we have unique solution for these equations.



$$\frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{(-7)(3)-(-15)(1)} = \frac{1}{(-3)1-(-3)3}$$

$$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$\Rightarrow x = 4$ and $y = -1$

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2. (i) For which values of a and b does the following pair of linear equations have an

infinite number of solutions?

$2x + 3y = 7$

$(a - b)\,x + (a + b)\,y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)\,x + (k - 1)\,y = 2k + 1$

Ans. (i) Comparing equation $2x + 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $(a - b)\,x + (a + b)\,y - 3a - b + 2 = 0$ with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 2, b_1 = 3$ and $c_1 = -7, a_2 = (a-b), b_2 = (a+b)$ and $c_2 = 2 - b - 3a$

Linear equations have infinite many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{2-b-3a}$$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} \text{ and } \frac{3}{a+b} = \frac{-7}{2-b-3a}$$

$\Rightarrow 2a + 2b = 3a - 3b$ and $6 - 3b - 9a = -7a - 7b$

$\Rightarrow a = 5b$... (1) and $-2a = -4b - 6$... (2)

Putting (1) in (2), we get

$-2\,(5b) = -4b - 6$

$\Rightarrow -10b + 4b = -6$

$\Rightarrow -6b = -6 \Rightarrow b = 1$

Putting value of b in (1), we get

$a = 5b = 5\ (1) = 5$

Therefore, $a = 5$ and $b = 1$

(ii) Comparing $(3x + y - 1 = 0)$ with $a_1x + b_1y + c_1 = 0$ and $(2k - 1)x + (k - 1)y - 2k - 1 = 0)$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = 1$ and $c_1 = -1, a_2 = (2k-1), b_2 = (k-1)$ and $c_2 = -2k - 1$

Linear equations have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-2k-1}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1}$

$\Rightarrow 3\ (k - 1) = 2k - 1$

$\Rightarrow 3k - 3 = 2k - 1$

$\Rightarrow k = 2$

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

Ans. Substitution Method

$8x + 5y = 9$ …**(1)**

$3x + 2y = 4$ …**(2)**

From equation **(1)**,

$5y = 9 - 8x \Rightarrow y = \frac{9-8x}{5}$

Putting this in equation **(2)**, we get

$3x + 2\left(\frac{9-8x}{5}\right) = 4$

$\Rightarrow 3x + \frac{18-16x}{5} = 4$

$\Rightarrow 3x - \frac{16}{5}x = \frac{4}{1} - \frac{18}{5}$

$\Rightarrow 15x - 16x = 20 - 18$

$\Rightarrow x = -2$

Putting value of **x** in **(1)**, we get

$8(-2) + 5y = 9$

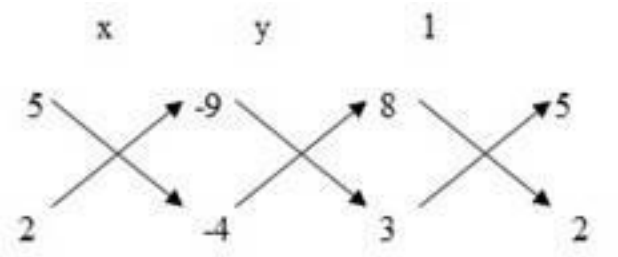
$\Rightarrow 5y = 9 + 16 = 25 \Rightarrow y = 5$

Therefore, $x = -2$ and $y = 5$

Cross multiplication method

$8x + 5y = 9$ … **(1)**

$3x + 2y = 4$ … **(2)**



$$\frac{x}{5(-4)-2(-9)}=\frac{y}{(-9)3-(-4)8}=\frac{1}{8\times 2-5\times 3}$$

$$\Rightarrow \frac{x}{-20+18}=\frac{y}{-27+32}=\frac{1}{16-15}$$

$$\Rightarrow \frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$

$\Rightarrow x = -2$ and $y = 5$

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4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the

rectangle.

Ans.(i) Let fixed monthly charge = Rs x and let charge of food for one day = Rs y

According to given conditions,

$x + 20y = 1000$ … (1),

and $x + 26y = 1180$ … (2)

Subtracting equation (1) from equation (2), we get

$6y = 180$

$\Rightarrow y = 30$

Putting value of y in (1), we get

$x + 20\ (30) = 1000$

$\Rightarrow x = 1000 - 600 = 400$

Therefore, fixed monthly charges = Rs 400 and, charges of food for one day = Rs 30

(ii) Let numerator = x and let denominator = y

According to given conditions,

$$\frac{x-1}{y}=\frac{1}{3} \quad ...(1) \qquad \frac{x}{y+8}=\frac{1}{4} \quad ...(2)$$

$\Rightarrow 3x - 3 = y$ ... (1) $4x = y + 8$ … (1)

$\Rightarrow 3x - y = 3$ … (1) $4x - y = 8$ … (2)

Subtracting equation (1) from (2), we get

$4x - y - (3x - y) = 8 - 3$

$\Rightarrow x = 5$

Putting value of x in (1), we get

3 (5) – y = 3

⇒ 15 – y = 3

⇒ y = 12

Therefore, numerator = 5 and, denominator = 12

It means fraction = $\frac{x}{y} = \frac{5}{12}$

(iii) Let number of correct answers = x and let number of wrong answers = y

According to given conditions,

3x – y = 40 …(1)

And, 4x − 2y = 50 … (2)

From equation (1), y = 3x − 40

Putting this in (2), we get

4x – 2 (3x − 40) = 50

⇒ 4x − 6x + 80 = 50

⇒ −2x = −30

⇒ x = 15

Putting value of x in (1), we get

3 (15) – y = 40

⇒ 45 – y = 40

⇒ y = 45 – 40 = 5

Therefore, number of correct answers = x = 15and number of wrong answers = y = 5

Total questions = x + y = 15 + 5 = 20

(iv) Let speed of car which starts from part A = x km/hr

Let speed of car which starts from part B = y km/hr

According to given conditions,

$\frac{100}{x-y}=5$ (Assuming x > y)

$\Rightarrow 5x - 5y = 100$

$\Rightarrow x - y = 20$ … (1)

And, $\frac{100}{x+y}=1$

$\Rightarrow x + y = 100$ … (2)

Adding (1) and (2), we get

$2x = 120$

$\Rightarrow x = 60$ km/hr

Putting value of x in (1), we get

$60 - y = 20$

$\Rightarrow y = 60 - 20 = 40$ km/hr

Therefore, speed of car starting from point A = 60 km/hr

And, Speed of car starting from point B = 40 km/hr

(v) Let length of rectangle = x units and Let breadth of rectangle = y units

Area = xy square $units$. According to given conditions,

$xy - 9 = (x - 5)(y + 3)$

$\Rightarrow xy - 9 = xy + 3x - 5y - 15$

$\Rightarrow 3x - 5y = 6$ … (1)

And, $xy + 67 = (x + 3)(y + 2)$

$\Rightarrow xy + 67 = xy + 2x + 3y + 6$

$\Rightarrow 2x + 3y = 61$ … (2)

From equation (1),

$3x = 6 + 5y$

$\Rightarrow x = \frac{6+5y}{3}$

Putting this in (2), we get

$2\left(\frac{6+5y}{3}\right) + 3y = 61$

$\Rightarrow 12 + 10y + 9y = 183$

$\Rightarrow 19y = 171$

$\Rightarrow y = 9$ units

Putting value of y in (2), we get

$2x + 3(9) = 61$

$\Rightarrow 2x = 61 - 27 = 34$

$\Rightarrow x = 17$ units

Therefore, length = 17 units and, breadth = 9 units