Exercise-2.5

1. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

(ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Ans.(i) $(x+4)(x+10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(x+4)(x+10)$

$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$

$=x^2+14x+40$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2+14x+40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(x+8)(x-10)$

$(x+8)(x-10)=x^2+[8+(-10)]x+[8\times(-10)]$

$=x^2-2x-80.$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2-2x-80$.

(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$(3x+4)(3x-5)=(3x)^2+[4+(-5)]3x+[4\times(-5)]$

$=9x^2-3x-20.$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2-3x-20$.

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$=(y^2)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}.$

Therefore, we conclude that the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ is $\left(y^4-\frac{9}{4}\right)$.

(v) $(3+2x)(3-2x)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $(3+2x)(3-2x)$

$(3+2x)(3-2x)=(3)^2-(2x)^2$

$=9-4x^2.$

Therefore, we conclude that the product $(3+2x)(3-2x)$ is $(9-4x^2)$.

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2. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98×96

(iii) 104×96

Ans. (i) 103×107

103×107 can also be written as $(100+3)(100+7)$.

We can observe that, we can apply the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

$(100+3)(100+7) = (100)^2 + (3+7)(100) + 3 \times 7$

=10000+1000+21

=11021

Therefore, we conclude that the value of the product 103×107 is 11021.

(ii) 95×96

95×96 can also be written as $(100-5)(100-4)$

We can observe that, we can apply the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

$(100-5)(100-4) = (100)^2 + [(-5)+(-4)](100) + (-5) \times (-4)$

=10000-900+20 =9120

Therefore, we conclude that the value of the product 95×96 is 9120.

(iii) 104×96

104×96 can also be written as $(100+4)(100-4)$.

We can observe that, we can apply the identity $(x+y)(x-y) = x^2 - y^2$ with respect to the expression $(100+4)(100-4)$, to get

$(100+4)(100-4) = (100)^2 - (4)^2$

=10000-16

=9984

Therefore, we conclude that the value of the product 104×96 is 9984.

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3. Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Ans. (i) $9x^2+6xy+y^2$

$9x^2+6xy+y^2=(3x)^2+2\times 3x\times y+(y)^2$

We can observe that, we can apply the identity $(x+y)^2=x^2+2xy+y^2$

$\Rightarrow (3x)^2+2\times 3x\times y+(y)^2=(3x+y)^2$.

$\Rightarrow (3x+y)(3x+y)$

(ii) $4y^2-4y+1$

$4y^2-4y+1=(2y)^2-2\times 2y\times 1+(1)^2$

We can observe that, we can apply the identity $(x-y)^2=x^2-2xy+y^2$

$\Rightarrow (2y)^2-2\times 2y\times 1+(1)^2=(2y-1)^2$.

$\Rightarrow (2y-1)(2y-1)$

(iii) $x^2-\frac{y^2}{100}$

We can observe that, we can apply the identity $(x)^2-(y)^2=(x+y)(x-y)$

$\Rightarrow (x)^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$.

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Ans. (i) $(x+2y+4z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(x+2y+4z)^2$.

$$(x+2y+4z)^2 = (x)^2+(2y)^2+(4z)^2+2\times x\times 2y+2\times 2y\times 4z+2\times 4z\times x$$
$$= x^2+4y^2+16z^2+4xy+16yz+8zx$$

(ii) $(2x-y+z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(2x-y+z)^2$.

$$(2x-y+z)^2 = [2x+(-y)+z]^2$$
$$= (2x)^2+(-y)^2+(z)^2+2\times 2x\times(-y)+2\times(-y)\times z+2\times z\times 2x$$
$$= 4x^2+y^2+z^2-4xy-2yz+4zx.$$

(iii) $(-2x+3y+2z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+3y+2z)^2$.

$$(-2x+3y+2z)^2 = [(-2x)+3y+2z]^2$$
$$= (-2x)^2+(3y)^2+(2z)^2+2\times(-2x)\times 3y+2\times 3y\times 2z+2\times 2z\times(-2x)$$
$$= 4x^2+9y^2+4z^2-12xy+12yz-8zx$$

(iv) $(3a-7b-c)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(3a-7b-c)^2$.

$$(3a-7b-c)^2 = [3a+(-7b)+(-c)]^2$$
$$= (3a)^2+(-7b)^2+(-c)^2+2\times 3a\times(-7b)+2\times(-7b)\times(-c)+2\times(-c)\times 3a$$
$$= 9a^2+49b^2+c^2-42ab+14bc-6ac.$$

(v) $(-2x+5y-3z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+5y-3z)^2$

$$(-2x+5y-3z)^2 = [(-2x)+5y+(-3z)]^2$$
$$=(-2x)^2+(5y)^2+(-3z)^2+2\times(-2x)\times 5y+2\times 5y\times(-3z)+2\times(-3z)\times(-2x)$$
$$=4x^2+25y^2+9z^2-20xy-30yz+12zx.$$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

$$\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \left[\frac{a}{4}+\left(-\frac{b}{2}\right)+1\right]^2$$
$$=\left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(1)^2+2\times\frac{a}{4}\times\left(-\frac{b}{2}\right)+2\times\left(-\frac{b}{2}\right)\times 1+2\times 1\times\frac{a}{4}$$
$$=\frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}.$$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Ans. (i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

The expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$ can also be written as

$$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x.$$

We can observe that, we can apply the identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression

$$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x,$$

We get $(2x+3y-4z)^2 = (2x+3y-4z)(2x+3y-4z)$

Therefore, we conclude that after factorizing the expression

$4x^2+9y^2+16z^2+12xy-24yz-16xz$, we get.

$(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

We need to factorize the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$.

The expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ can also be written as

$\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$.

We can observe that, we can apply the identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression $\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$, to get

$\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$

Therefore, we conclude that after factorizing the expression

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$,

we get $(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans. (i) $(2x+1)^3$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.

$$\therefore (2x+1)^3=(2x)^3+(1)^3+3\times 2x\times 1(2x+1)$$
$$=8x^3+1+6x(2x+1)$$
$$=8x^3+12x^2+6x+1.$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3+12x^2+6x+1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\therefore (2a-3b)^3 = (2a)^3 - (3b)^3 - 3\times 2a \times 3b(2a-3b)$$
$$= 8a^3 - 27b^3 - 18ab(2a-3b)$$
$$= 8a^3 - 36a^2b + 54ab^2 - 27b^3.$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3-36a^2b+54ab^2-27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times 1\left(\frac{3}{2}x+1\right)$$
$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$
$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\therefore \left(x-\frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3.$

Therefore, the expansion of the expression $\left(x-\frac{2}{3}y\right)^3$ is $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$.

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7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans. (i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Usingidentity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(100-1)^3 = (100)^3 - (1)^3 - 3\times 100\times 1(100-1)$

=1000000-1-300(99)

=999999-29700

=970299

(ii) $(102)^3$

$(102)^3$ can also be written as $(100+2)^3$.

Usingidentity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(100+2)^3 = (100)^3 + (2)^3 + 3\times 100\times 2(100+2)$

=1000000+8+600(102)

=1000008+61200

=1061208

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(1000-2)^3 = (1000)^3 - (2)^3 - 3\times1000\times2(1000-2)$

=1000000000-8-6000(998)

=999999992-5988000

=994011992

8. Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 + (b)^3 + 3\times2a\times2a\times b + 3\times2a\times b\times b$

$= (2a)^3 + (b)^3 + 3\times2a\times b(2a+b)$.

Using identity

$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$ with respect to the expression $(2a)^3 + (b)^3 + 3\times2a\times b(2a+b)$, we get $(2a+b)^3 = (2a+b)(2a+b)(2a+b)$

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a+b)(2a+b)(2a+b)$.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b)$, we get $(2a-b)^3 = (2a-b)(2a-b)(2a-b)$

Therefore, after factorizing the expression

$8a^3 - b^3 - 12a^2b + 6ab^2$,

we get $(2a-b)(2a-b)(2a-b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ can also be written as

$= (3)^3 - (5a)^3 - 3 \times 3 \times 3 \times 5a + 3 \times 3 \times 5a \times 5a$

$= (3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a).$

Using identity

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression

$27 - 125a^3 - 135a + 225a^2$,we get $(3-5a)^3 = (3-5a)(3-5a)(3-5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$= (4a)^3 - (3b)^3 - 3 \times 4a \times 4a \times 3b + 3 \times 4a \times 3b \times 3b$

$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

with respect to the expression

$(4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$,

we get $(4a-3b)^3 = (4a-3b)(4a-3b)(4a-3b)$

Therefore, after factorizing the expression

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

we get $(4a-3b)(4a-3b)(4a-3b)$.

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

The expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ can also be written as

$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times3p\times\frac{1}{6}+3\times3p\times\frac{1}{6}\times\frac{1}{6}$

$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$

with respect to the expression $(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right)$,

to get $\left(3p-\frac{1}{6}\right)^3=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

Therefore, after factorizing the expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$,

we get $\left(3p-\frac{1}{6}\right)^3=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

9. Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Ans. (i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.

$\Rightarrow x^3+y^3=(x+y)^3-3xy(x+y)$

$=(x+y)\left[(x+y)^2-3xy\right]$

$\because$ We know that $(x+y)^2=x^2+2xy+y^2$

$\therefore\ x^3+y^3=(x+y)(x^2+2xy+y^2-3xy)$

$= (x+y)(x^2 - xy + y^2)$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$= (x-y)\left[(x-y)^2 + 3xy\right]$

$\because$ We know that $(x-y)^2 = x^2 - 2xy + y^2$

$\therefore\ x^3 - y^3 = (x-y)(x^2 - 2xy + y^2 + 3xy)$

$= (x-y)(x^2 + xy + y^2)$

Therefore, the desired result has been verified.

10. Factorize:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Ans. (i) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$.

$(3y)^3 + (5z)^3 = (3y+5z)\left[(3y)^2 - 3y \times 5z + (5z)^2\right]$

$= (3y+5z)(9y^2 - 15yz + 25z^2)$.

(ii) $64m^3 - 343n^3$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$.

$(4m)^3 - (7n)^3 = (4m-7n)\left[(4m)^2 + 4m \times 7n + (7n)^2\right]$.

$= (4m-7n)(16m^2 + 28mn + 49n^2)$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m-7n)(16m^2+28mn+49n^2)$.

11. Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Ans. The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as $(3x)^3 + (y)^3 + (z)^3 - 3\times 3x \times y \times z.$

We know that $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

$$\therefore (3x)^3 + (y)^3 + (z)^3 - 3\times 3x\times y\times z = (3x+y+z)\left[(3x)^2+(y)^2+(z)^2-3x\times y-y\times z-z\times 3x\right]$$

$$= (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz).$$

Therefore, we conclude that after factorizing the expression $27x^3 + y^3 + z^3 - 9xyz$, we get $(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$.

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

Ans.

LHS is $x^3 + y^3 + z^3 - 3xyz$ and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

And also, we know that $(x-y)^2 = x^2 - 2xy + y^2$.

$$\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

$$\frac{1}{2}(x+y+z)\left[(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2xz+x^2)\right]$$

$$\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

Therefore, we can conclude that the desired result is verified.

13. If $x+y+z=0$ **, show that** $x^3+y^3+z^3=3xyz$

Ans. We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

We need to substitute x + y + z = 0

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$, to get

$x^3+y^3+z^3-3xy=(0)\times(x^2+y^2+z^2-xy-yz-zx)$

$x^3+y^3+z^3-3xyz=0$

$\Rightarrow x^3+y^3+z^3=3xyz.$

Therefore, the desired result is verified.

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Ans. (i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12$, $b=7$ and $c=5$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$\therefore(-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$=-1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28$, $b=-15$ and $c=-13$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

= 16380

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2-35a+12$

(ii) Area : $35y^2+13y-12$

Ans. (i) Area : $25a^2-35a+12$

The expression $25a^2-35a+12$ can also be written as $25a^2-15a-20a+12$.

$25a^2-15a-20a+12=5a(5a-3)-4(5a-3)$

$=(5a-4)(5a-3)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2-35a+12$ is Length $=(5a-4)$ and Breadth $=(5a-3)$.

(ii) Area : $35y^2+13y-12$

The expression $35y^2+13y-12$ can also be written as $35y^2+28y-15y-12$.

$35y^2+28y-15y-12=7y(5y+4)-3(5y+4)$

$=(7y-3)(5y+4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2+13y-12$ is Length $=(7y-3)$ and Breadth $=(5y+4)$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) $Volume: 3x^2 - 12x$

(ii) $Volume: 12ky^2 + 8ky - 20k$

Ans. (i) Volume : $3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x-4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is $3, x$ and $(x-4)$ .so we get length=3,breadth=x, height=(x-4)

(ii) Volume : $12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$k(12y^2 + 8y - 20) = k(12y^2 - 12y + 20y - 20)$

$= k[12y(y-1) + 20(y-1)]$

$= k(12y + 20)(y-1)$

$= 4k \times (3y+5) \times (y-1)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k, (3y+5)$ and $(y-1)$.

so we ger length=4k, breadth= 3y+5, height= y-1