Exercise 13.5

1. A copper wire, 3 mm in diameter is wound about a cylinder whose length is 12 cm and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3 .

Ans. Number of rounds to cover 12 cm, i.e. 120 mm = $\frac{120}{3}$ = 40

Here, Diameter = 10 cm, Radius $(r) = \frac{10}{2}$ cm

Length of the wire used in taking one round

= $2\pi r = 2\pi \times 5 = 10\pi$ cm

Length of the wire used in taking 40 rounds

= $10\pi \times 40 = 400\pi$ cm

Radius of the copper wire = $\frac{3}{2}$ mm

= $\frac{3}{20}$ cm

∴ Volume of wire = $\pi\left(\frac{3}{20}\right)^2(400\pi)$

= $9\pi^2 cm^3$--------------***

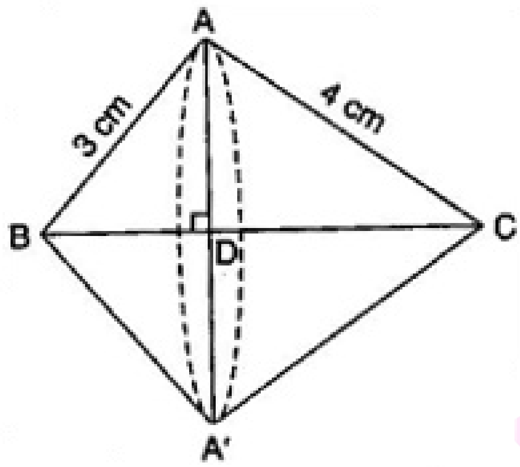
∴ Mass of the wire = $9\times(3.14)^2\times 8.88$

= 787.98 gm

2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate)

Ans. Hypotenuse = $\sqrt{3^2 + 4^2}$ = 5 cm

In figure, $\triangle$ ADB $\sim$ $\triangle$ CAB [AA similarity]



$\therefore \frac{AD}{CA} = \frac{AB}{CB}$

$\Rightarrow \frac{AD}{4} = \frac{3}{5}$

$\Rightarrow$ AD = $\frac{12}{5}$ cm

Also, $\frac{DB}{AB} = \frac{AB}{CB}$

$\Rightarrow \frac{DB}{3} = \frac{3}{5}$

$\Rightarrow$ DB = $\frac{9}{5}$ cm

$\therefore$ CD = BC – DB = $5-\frac{9}{5}=\frac{16}{5}$ cm

Volume of the double cone

= $\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{9}{5}\right)+\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{16}{5}\right)$

= $\frac{1}{3}\times 3.14\times\frac{12}{5}\times\frac{12}{5}\times 5$ = $30.14\ cm^3$

Surface area of the double cone

= $\pi\times\frac{12}{5}\times 3+\pi\times\frac{12}{5}\times 4$

= $\pi\times\frac{12}{5}(3+4)$ = $3.14\times\frac{12}{5}\times 7$

= $52.75\ cm^2$

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3. A cistern, internally measuring $150\text{ cm}\times 120\text{ cm}\times 110\text{ cm}$ has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being $22.5\text{ cm}\times 7.5\text{ cm}\times 6.5\text{ cm}$?

Ans. Volume of cistern = $150\times 120\times 110$ = $1980000\ cm^3$

Volume of water = $129600\ cm^3$

$\therefore$ Volume of cistern to be filled

= 1980000 – 129600 = $1850400\ cm^3$

Volume of a brick = $22.5\times 7.5\times 6.5$

= $1096.875\ cm^3$

Let n bricks be needed.

Then, water absorbed by n bricks = $n \times \frac{1096.875}{17} cm^3$

$\therefore \ n = \frac{1850400 \times 17}{16 \times 1096.875}$ = 1792 (approx.)

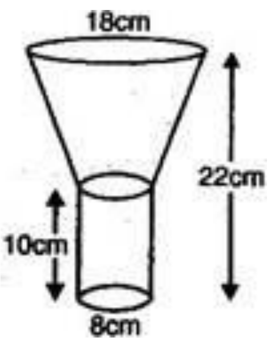
4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Ans. Volume of rainfall = $7280 \times \frac{10}{100 \times 1000}$ = 0.728 km^2

Volume of three rivers = $3 \times 1072 \times \frac{75}{1000} \times \frac{3}{1000}$ = 0.7236 km^2

Hence, the amount of rainfall is approximately equal to the amount of water in three rivers.

5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see figure).



Ans. Slant height of the frustum of the cone

$$(l) = \sqrt{h^2 + (r_1 - r_2)^2}$$

= $\sqrt{(22-10)^2+\left(\frac{18}{2}-\frac{8}{2}\right)^2}$ = 13 cm

Area of the tin sheet required

= CSA of cylinder + CSA of the frustum

= $2\pi(4)(10)+\pi(4+9)13$

= $80\pi+169\pi$

= 249π = $249\times\frac{22}{7}$ = $782\frac{4}{7}cm^2$

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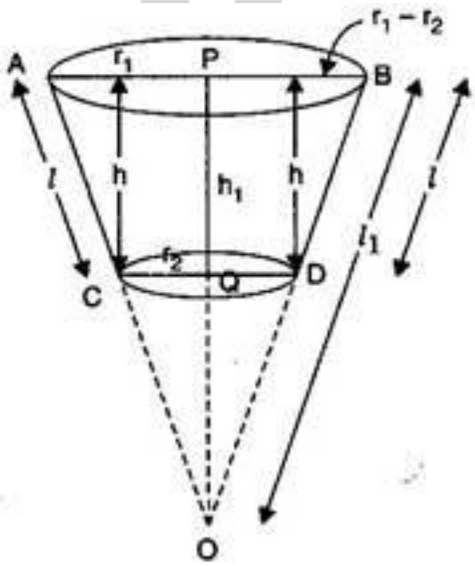
6. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. According to the question, the frustum is the difference of the two cones OAB and OCD (in figure).



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

∴ Height of the cone OCD = $h_1 - h$

$\because$ Δ OQD ~ Δ OPB [By, AA similarity]

$$\therefore \frac{h_1 - h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow 1 - \frac{r_2}{r_1} = \frac{h}{h_1}$$

$$\Rightarrow h_1 = \frac{hr_1}{r_1 - r_2} \quad \text{..........(i)}$$

$\therefore$ height of the cone OCD = $h_1 - h$

$$= \frac{hr_1}{r_1 - r_2} - h = \frac{hr_2}{r_1 - r_2} \quad \text{..........(ii)}$$

$\therefore$ V of the frustum

= V of cone OAB – V of cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$= \frac{\pi}{3}\left[r_1^2 . \frac{hr_1}{r_1 - r_2} - r_2^2 . \frac{hr_2}{r_1 - r_2}\right]$$

[From eq. (i) & (ii)]

$$= \frac{\pi h}{3}\left(\frac{r_1^3 - r_2^3}{r_1 - r_2}\right)$$

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

If $A_1 and A_2$ are the surface areas of two circular bases, then

$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

$\therefore$ V of the frustum

$$= \frac{h}{3}\left(\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2} . \sqrt{\pi r_2^2}\right)$$

$$= \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

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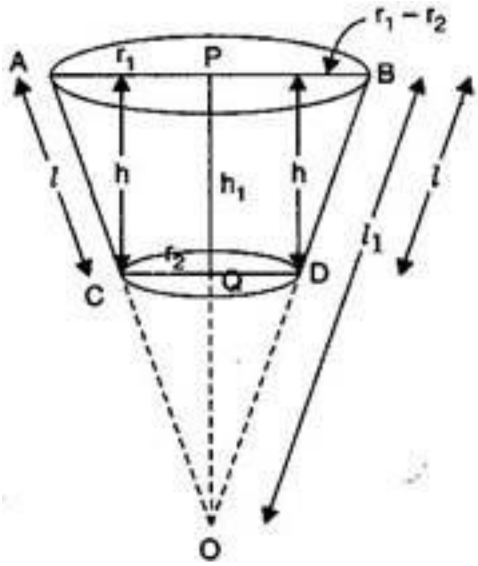
7. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans.



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

Again, from ΔDEB, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\because$ Δ OQD ~ Δ OPB [AA similarity]

$$\therefore \frac{l_1 - l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow l_1 = \frac{lr_1}{r_1 - r_2} \quad \text{..........(iii)}$$

$$\therefore l_1 - l = \frac{lr_1}{r_1 - r_2} - l = \frac{lr_2}{r_1 - r_2} \quad \text{.............................. (iv)}$$

Hence, CSA of the frustum of the cone = $\pi r_1 l_1 - \pi r_2 (l_1 - l)$

$$= \pi r_1 . \frac{lr_1}{r_1 - r_2} - \pi r_2 \frac{lr_2}{r_1 - r_2} \quad \text{[From eq. (i) and (ii)]}$$

$$= \pi l \left(\frac{r_1^2 - r_2^2}{r_1 - r_2} \right) = \pi l (r_1 + r_2),$$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\therefore$ TSA of the frustum of the cone

$$= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$