Exercise 13.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

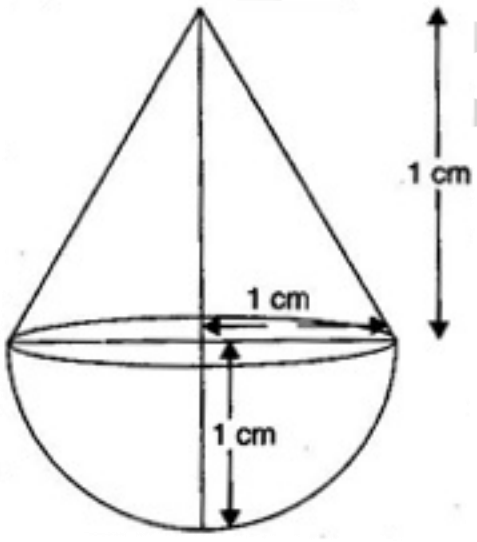
1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Ans. For hemisphere, Radius (r) = 1 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(1)^3$

= $\frac{2}{3}\pi$ cm^3



For cone, Radius of the base (r) = 1 cm

Height (h) = 1 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(1)^2 \times 1$

= $\frac{1}{3}\pi$ cm^3

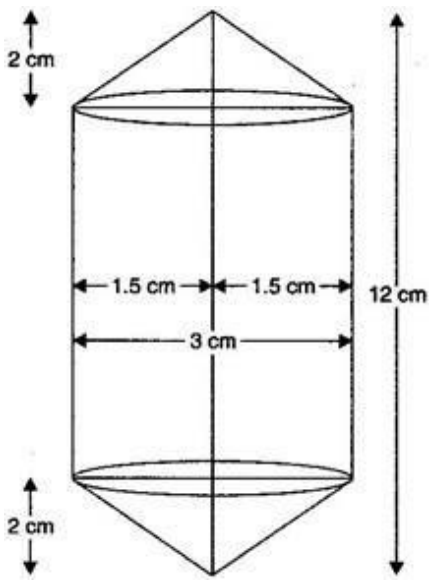
∴ Volume of the solid = Volume of hemisphere + Volume of cone

= $\frac{2}{3}\pi + \frac{1}{3}\pi = \pi$ cm^3

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Ans. For upper conical portion, Radius of the base (r) = 1.5 cm

Height (h_1) = 2 cm



Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(1.5)^2 \times 2$

= $1.5\pi cm^3$

For lower conical portion, Volume = $1.5\pi cm^3$

For central cylindrical portion:

Radius of the base (r) = 1.5 cm

Height (h_2) = 12 – (2 + 2) = 8 cm

Volume = $\pi r^2 h_2 = \pi(1.5)^2 \times 8 = 18\pi cm^3$

∴ Volume of the model = $1.5\pi + 1.5\pi + 18\pi$ = volume of top cone + volume of bottom cone + volume of cylindrical part

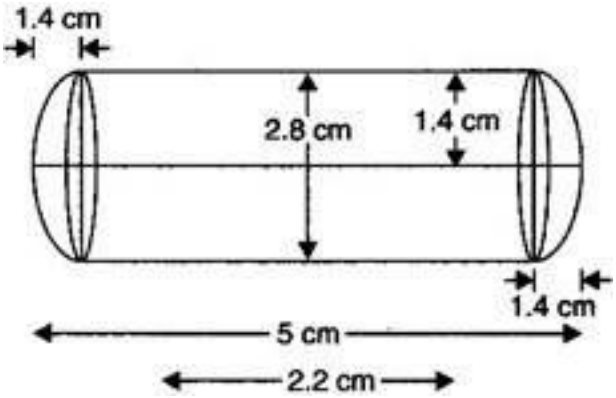
21π

$= 21 \times \frac{22}{7} = 66\ cm^3$

3. A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends, with length 5 cm and diameter 2.8 cm (see figure).

Ans. Volume of a gulab jamun = $\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$ = volume of 2hemisphere + volume of cylinder

$= \frac{2}{3}\pi(1.4)^3 + \pi(1.4)^2 \times 2.2 + \frac{2}{3}\pi(1.4)^3$



$= \frac{4}{3}\pi(1.4)^3 + \pi(1.4)^2 \times 2.2$

$= \pi(1.4)^2\left[\frac{4 \times 1.4}{3} + 2.2\right]$

$= \pi \times 1.96\left[\frac{5.6+6.6}{3}\right] = \frac{1.96 \times 12.2}{3}\pi cm^3$

$\therefore$ Volume of 45 gulab jamuns

$= 45 \times \frac{1.96 \times 12.2}{3}\pi$

$= 15 \times 1.96 \times 12.2 \times \frac{22}{7}$

$= 1127.28\ cm^3$

$\therefore$ Volume of syrup = $1127.28 \times \frac{30}{100}$ = 30% of volume of 45 gulab jamun

$= 338.184\ cm^3 = 338\ cm^3$ (approx.)

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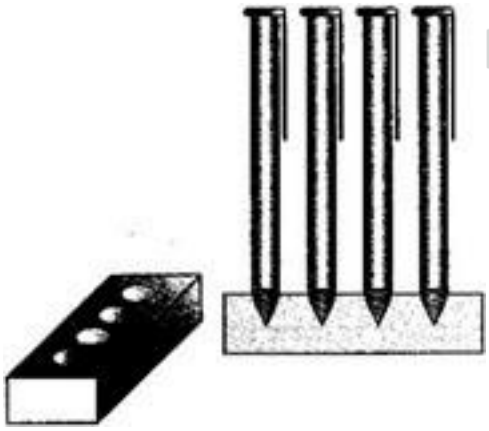
4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth id 1.4 cm. Find the volume of wood in the entire stand (see figure).



Ans: For Cuboid:

l=15 cm

b=10 cm

h=3.5 cm

Volume of the cuboid = $l \times b \times h$

= $15 \times 10 \times 3.5$

= $525\ cm^3$

For Cone: r = 0.5 cm

h = 1.4 cm

Volume of conical depression = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

= $\frac{11}{30} cm^3$

∴ Volume of four conical depressions = $4 \times \frac{11}{30} = 1.47\ cm^3$

∴ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depression = $525 - 1.47 = 523.53\ cm^3$

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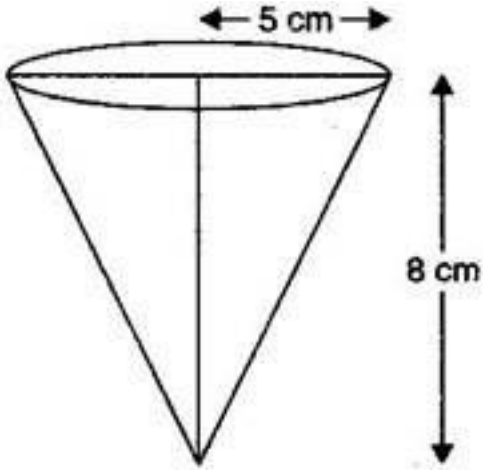
5. A vessel is in the form of inverted cone. Its height is 8 cm and the radius of the top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Ans. For cone, Radius of the top (r) = 5 cm and height (h) = 8 cm



Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(5)^2 \times 8$

$= \frac{200}{3}\pi \text{ cm}^3$

For spherical lead shot, Radius (R) = 0.5 cm

Volume of spherical lead shot = $\frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi(0.5)^3$

$= \frac{\pi}{6} cm^3$

Volume of water that flows out = $\frac{1}{4}$ Volume of the cone

$= \frac{1}{4} \times \frac{200\pi}{3} = \frac{50\pi}{3} cm^3$

Let the number of lead shots dropped in the vessel be n.

n * volume of spherical shot = volume of water flows out

$\therefore\ n \times \frac{\pi}{6} = \frac{50\pi}{3}$

$\Rightarrow n = \frac{50\pi}{3} \times \frac{6}{\pi}$

$\Rightarrow n = 100$

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6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that $1\ cm^3$ of iron has approximately 8 g mass.

$(\text{Use } \pi = 3.14)$

Ans. For lower cylinder, Base radius $(r) = \frac{24}{2} = 12$ cm

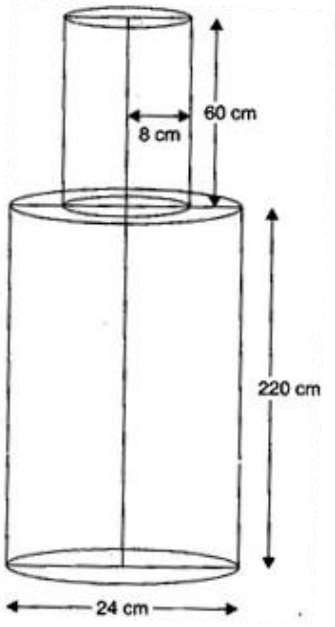
And Height $(h) = 220$ cm

Volume = $\pi r^2 h$

$= \pi(12)^2 \times 220$

$= 31680\pi\ cm^3$

For upper cylinder, Base Radius (R) = 8 cm



And Height (H) = 60 cm

Volume = $\pi R^2 H$

= $\pi(8)^2 \times 60$

= $3840\pi \ cm^3$

∴ Volume of the solid Iron pole

= V of lower cylinder + V of upper cylinder

= $31680\pi + 3840\pi = 35520\pi$

= $35520 \times 3.14 = 111532.8 \ cm^3$

mass of 1 cm^3 iron = 8 gm

mass of 111532.8 cm^3 iron = 8* 111532.8 = 892262.4 gm = 892.2624 kg

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Ans. For right circular cone, Radius of the base (r) = 60 cm

And Height (h_1) = 120 cm

Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(60)^2 \times 120$

= $144000\pi \ cm^3$

For Hemisphere, Radius of the base (r) = 60 cm

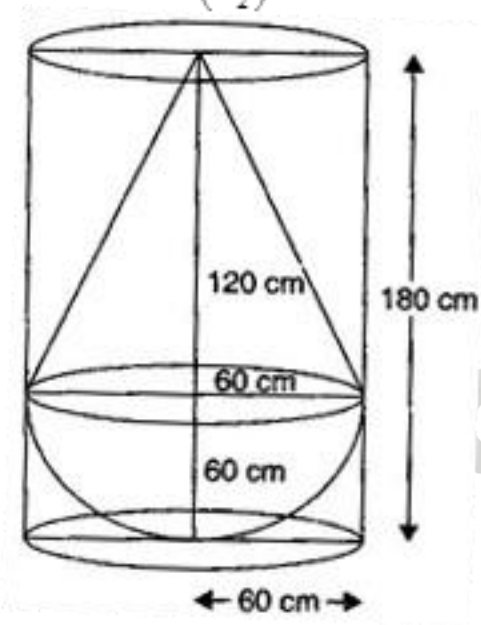
Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(60)^3$

= $144000\pi cm^3$

For right circular cylinder, Radius of the base (r) = 60 cm

And Height (h_2) = 180 cm



Volume = $\pi r^2 h_2$

= $\pi(60)^2 \times 180$

= $648000\pi\ cm^3$

Now, V of water left in the cylinder

= V of right circular cylinder – (V of right circular cone + V of hemisphere)

$= 648000\pi - (144000\pi + 144000\pi)$

$= 360000\pi \ cm^3$

$= \frac{360000}{100\times100\times100}\pi \ m^3$

$= 0.36\times\frac{22}{7} = 1.131 \ m^3$ (approx.)

7. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be $345 \ cm^3$. Check whether she is correct, taking the above as the inside measurements and $\pi = 3.14$.

Ans.

For Cylinder: diameter of cylin. = 2 cm, height of cylin. = 8 cm

For Sphere : diameter of sphere = 8.5 cm

Amount of water it holds = $\frac{4}{3}\pi r^3 + \pi r^2 h$ = volume of sphere + volume of cylinder

$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi\left(\frac{2}{2}\right)^2\times8$

$= \frac{4}{3}\times3.14\times4.25\times4.25\times4.25 + 8\times3.14$

$= 321.39 + 25.12$

$= 346.51 \ cm^3$

Hence, she is not correct. The correct volume is $346.51 \ cm^3$