## Exercise 10.2

**In Q 1 to 3, choose the correct option and give justification.**

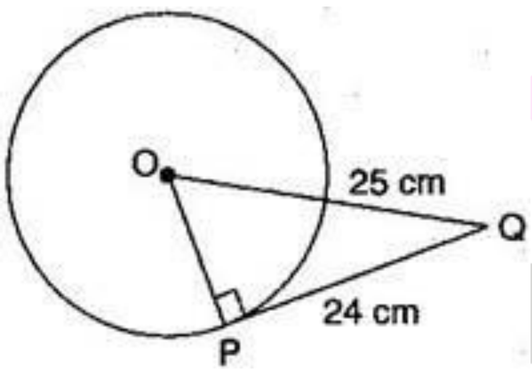
**1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:**

**(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm**

**Ans. (A)**

$\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]




$\therefore$ In right triangle OPQ,

$$OQ^2 = OP^2 + PQ^2$$

[By Pythagoras theorem]

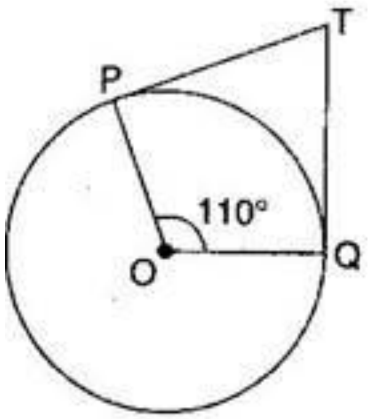
$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

$\Rightarrow OP^2 = 625 - 576 = 49$

$\Rightarrow$ OP = 7 cm

**2. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$POQ = $110^\circ$, then $\angle$PTQ is equal to:**




**(A)** $60^\circ$ **(B)** $70^\circ$ **(C)** $80^\circ$ **(D)** $90^\circ$

**Ans. (B)**

$\angle POQ = 110^\circ$, $\angle OPT = 90^\circ$ and $\angle OQT = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In quadrilateral OPTQ,

$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360^\circ$

[Angle sum property of quadrilateral]

$\Rightarrow 110^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow 290^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 360^\circ - 290^\circ$

$\Rightarrow \angle PTQ = 70^\circ$

**3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of $80^\circ$, then $\angle$POA is equal to:**

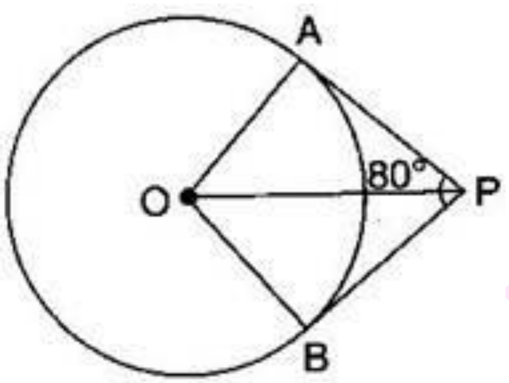
**(A) $50^\circ$ (B) $60^\circ$ (C) $70^\circ$ (D) $80^\circ$**

**Ans. (A)**

$\because \angle OAP = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]




$\angle OPA = \frac{1}{2}\angle BPA = \frac{1}{2} \times 80^\circ = 40^\circ$

[Centre lies on the bisector of the

angle between the two tangents]

In $\Delta OPA$,

$\angle OAP + \angle OPA + \angle POA = 180^\circ$

[Angle sum property of a triangle]

$\Rightarrow 90^\circ + 40^\circ + \angle POA = 180^\circ$

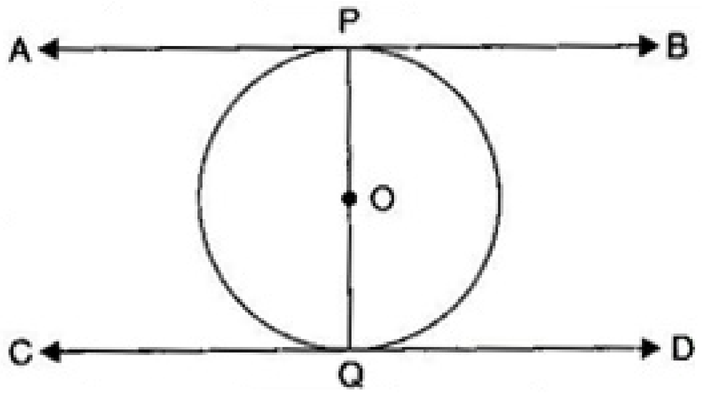
$\Rightarrow 130^\circ + \angle POA = 180^\circ$

$\Rightarrow \angle POA = 50^\circ$

**4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**

**Ans. Given**: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.




**To Prove**: AB $\parallel$ CD

**Proof**: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle OPA = 90^\circ$ .........(i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle OQD = 90^\circ$ .........(ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

$\therefore$ AB | | CD

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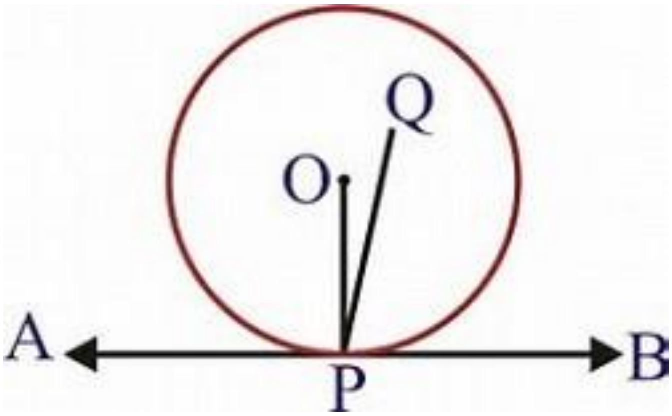
**5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.**

**Ans.** Let AB be the tangent drawn at the point P on the circle with O.




If possible, let PQ be perpendicular to AB, not passing through O.

Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, $AB \perp OP \quad \Rightarrow \quad \angle OPB = 90^{o}$

Also, $\angle QPB = 90^{o}$ [By construction]

Therefore, $\angle QPB = \angle OPB$, which is not possible as a part cannot be equal to whole.

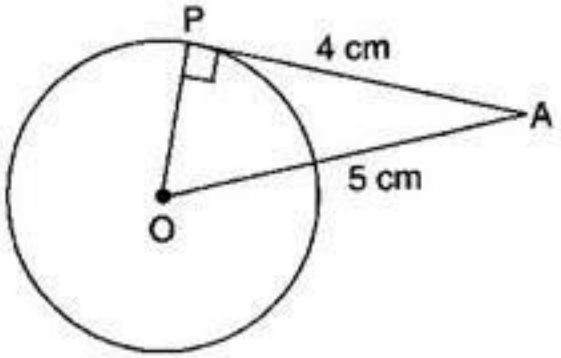
Thus, it contradicts our supposition.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.



**6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.**

**Ans.** We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.




$\therefore \angle OPA = 90^\circ$

$\therefore OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (OP)^2 + (4)^2$

$\Rightarrow 25 = (OP)^2 + 16$

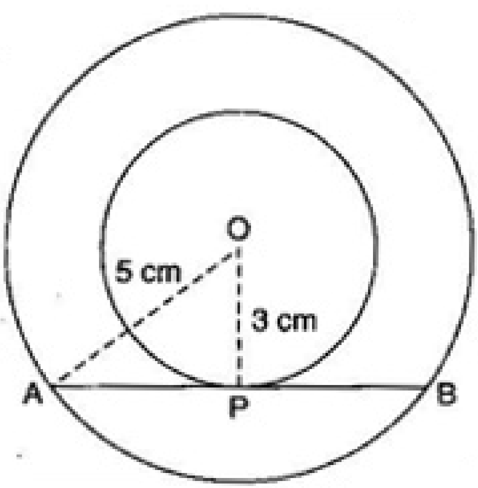
$\Rightarrow OP^2 = 9$

$\Rightarrow$ OP = 3 cm



**7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

**Ans.** Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = $90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = 4cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

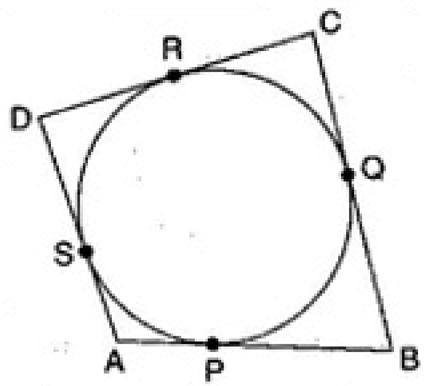
$\Rightarrow$ AB = AP + BP

= AP + AP = 2AP

= $2 \times 4$ = 8 cm

**8. A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:**

**AB + CD = AD + BC**




**Ans.** We know that the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ...........(i)

BP = BQ ........... (ii)

CR = CQ........... (iii)

DR = DS .......... (iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

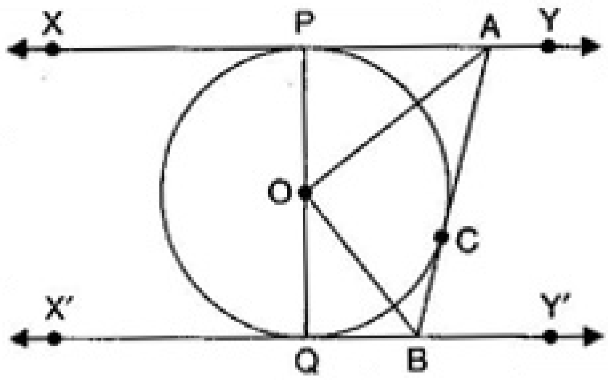
= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC



**9. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle$ AOB = $90^\circ$.**

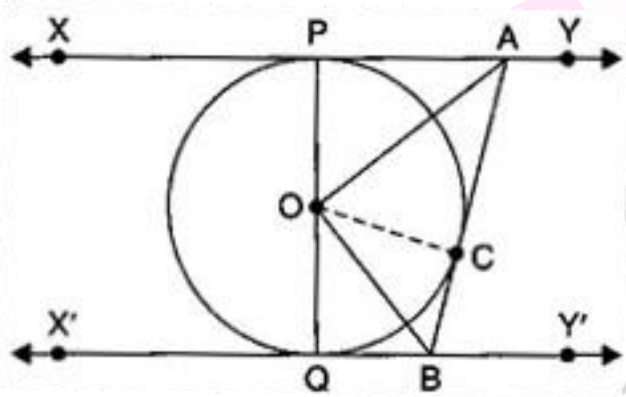



**Ans. Given**: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

**To Prove**: $\angle AOB = 90^\circ$

**Construction**: Join OC

**Proof**: $\angle OPA = 90^\circ$ ……….(i)




$\angle OCA = 90^\circ$ ……….(ii)

[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In right angled triangles OPA and OCA,

$\angle OPA = \angle OCA = 90^\circ$

OA = OA [Common]

AP = AC [Tangents from an external

point to a circle are equal]

$\therefore \Delta OPA \cong \Delta OCA$

[RHS congruence criterion]

$\therefore \angle OAP = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \angle OAC = \frac{1}{2}\angle PAB$............(iii)

Similarly, $\angle OBQ = \angle OBC$

$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$........... (iv)

$\because XY \parallel X'Y'$ and a transversal AB intersects them.

$\therefore \angle PAB + \angle QBA = 180^\circ$

[Sum of the consecutive interior angles on the same side of the transversal is $180^\circ$]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA$

$= \frac{1}{2} \times 180^\circ$........... (v)

$\Rightarrow \angle OAC + \angle OBC = 90^\circ$

[From eq. (iii) & (iv)]

In $\Delta AOB$,

$\angle OAC + \angle OBC + \angle AOB = 180^\circ$

[Angel sum property of a triangle]

$\Rightarrow 90^\circ + \angle AOB = 180^\circ$ [From eq. (v)]

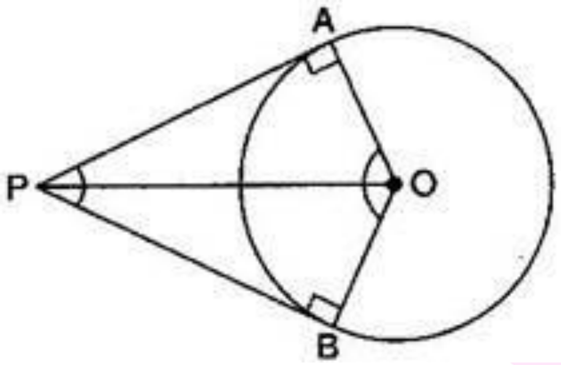
$\Rightarrow \angle AOB = 90^\circ$

Hence proved.

**10. Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.**

**Ans.** $\angle$ OAP = $90^\circ$ ……….(i)

$\angle$ OBP = $90^\circ$ ……….(ii)




[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore$ $\angle$ APB + $\angle$ AOB + $\angle$ OAP + $\angle$ OBP = $360^\circ$

[Angle sum property of a quadrilateral]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB + $90^\circ$ + $90^\circ$ = $360^\circ$

[From eq. (i) & (ii)]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB = $180^\circ$

$\therefore$ $\angle$ APB and $\angle$ AOB are supplementary.

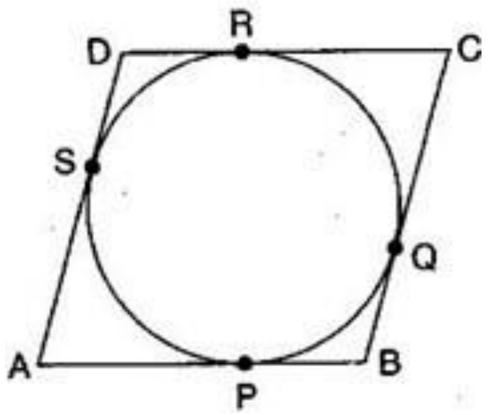
**11. Prove that the parallelogram circumscribing a circle is a rhombus.**

**Ans. Given**: ABCD is a parallelogram circumscribing a circle.

**To Prove**: ABCD is a rhombus.

**Proof**: Since, the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ...........(i)




BP = BQ ........... (ii)

CR = CQ........... (iii)

DR = DS .......... (iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD

[Opposite sides of $\parallel$gm are equal]

$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB = AD

But AB = CD and AD = BC

[Opposite sides of || gm]

∴ AB = BC = CD = AD

∴ Parallelogram ABCD is a rhombus.

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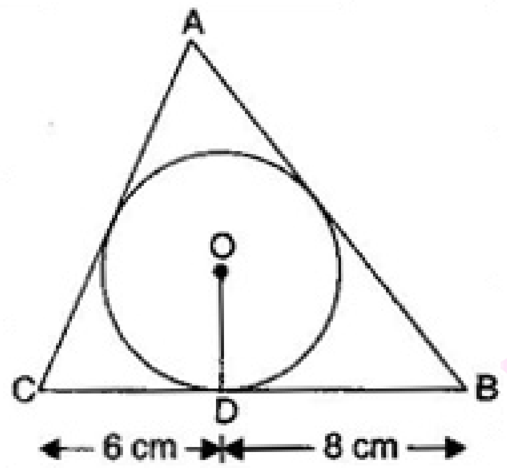
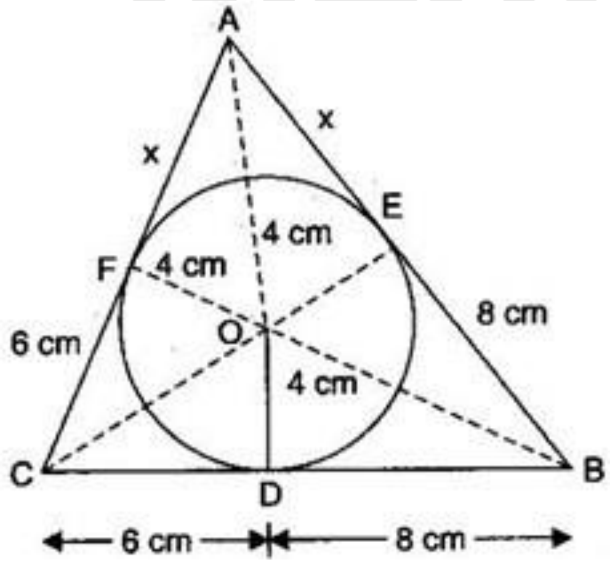
**12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.**




**Ans.** Join OE and OF. Also join OA, OB and OC.




Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = $x$

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

$\therefore$ Semi-perimeter of ABC $= \frac{(x+6)+(x+8)+(6+8)}{2} = (x+14)$ cm

$\therefore$ Area of $\Delta$ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

$= \sqrt{(x+14)(x)(6)(8)}$ cm$^2$

Now, Area of $\Delta$ABC = Area of $\Delta$OBC + Area of $\Delta$OCA + Area of $\Delta$OAB



$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= 28 + 2x + 12 + 2x + 16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4x + 56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14)$

Squaring both sides,

$$(x+14)(x)(6)(8)=16(x+14)^2$$

$$\Rightarrow 3x=x+14$$

$$\Rightarrow 2x=14$$

$$\Rightarrow x=7$$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

And AC = $x+6$ = 7 + 6 = 13 cm

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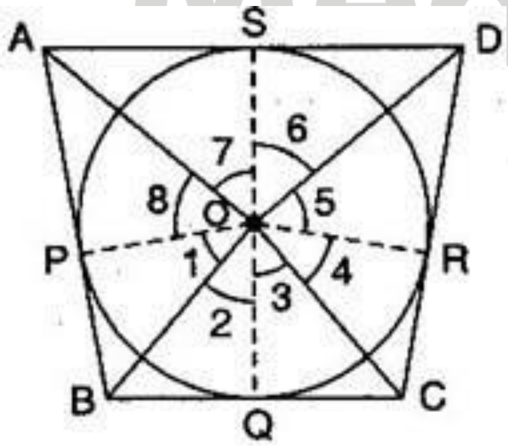
**13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**

**Ans.** Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = $180°$ (ii) $\angle$ BOC + $\angle$ AOD = $180°$

Construction: Join OP, OQ, OR and OS.




Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ ........... (i)

CQ = CR

DR = DS

In $\Delta$OBP and $\Delta$OBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore$ $\Delta$OPB $\cong$ $\Delta$OBQ [By SSS congruence criterion]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360^\circ$.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^\circ$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180^\circ$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180^\circ$

$\Rightarrow \angle$ AOB+ $\angle$ COD = $180^\circ$

Similarly, we can prove that

$\angle$ BOC + $\angle$ AOD = $180^\circ$