1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating decimal expansion.

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 \times 5^2}$

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

(viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$

(x) $\frac{77}{210}$

Ans. According to Theorem, any given rational number of the form $\frac{p}{q}$ **where *p* and *q* are co-prime**, has a terminating decimal expansion if q is of the form $2^n \times 5^m$, where m and n are non-negative integers.

(i) $\frac{13}{3125}$

$q = 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 2^0 \times 5^5$

Here, denominator is of the form $2^n \times 5^m$, where m = 5 and n = 0.

It means rational number $\frac{13}{3125}$ has a **terminating** decimal expansion.

(ii) $\frac{17}{8}$

$q = 8 = 2 \times 2 \times 2 = 2^3 = 2^3 \times 5^0$

Here, denominator is of the form $2^n \times 5^m$, where m = 0 and n = 3.

It means rational number $\frac{17}{8}$ has a **terminating** decimal expansion.

(iii) $\frac{64}{455}$

$q = 455 = 5 \times 91$

Here, denominator is not of the $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{64}{455}$ has a **non-terminating repeating** decimal expansion.

(iv) $\frac{15}{1600} = \frac{3}{320}$

$q = 320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 6.

It means rational number $\frac{15}{1600}$ has a **terminating** decimal expansion.

(v) $\frac{29}{343}$

$q = 343 = 7 \times 7 \times 7 = 7^3$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{29}{343}$ has **non-terminating repeating** decimal expansion.

(vi) $\frac{23}{2^3 \times 5^2}$

$q = 2^3 \times 5^2$

Here, denominator is of the form $2^n \times 5^m$, where m = 2 and n = 3 are non-negative integers.

It means rational number $\frac{23}{2^3 \times 5^2}$ has **terminating** decimal expansion.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

$q = 2^2 \times 5^7 \times 7^5$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{129}{2^2 \times 5^7 \times 7^5}$ has **non-terminating repeating** decimal expansion.

(viii) $\frac{6}{15} = \frac{2}{5}$

$q = 5 = 5^1 = 2^0 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 0.

It means rational number $\frac{6}{15}$ has **terminating** decimal expansion.

(ix) $\frac{35}{50} = \frac{7}{10}$

$q = 10 = 2 \times 5 = 2^1 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 1.

It means rational number $\frac{35}{50}$ has **terminating decimal** expansion.

(x) $\frac{77}{210} = \frac{11}{30}$

$q = 30 = 5 \times 3 \times 2$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{77}{210}$ has **non-terminating repeating** decimal expansion.

2. Write down the decimal expansions of those rational numbers in Question 1 which have terminating decimal expansions.

Ans. (i) $\frac{13}{3125} = \frac{13}{5^5} = \frac{13\times 2^5}{5^5\times 2^5} = \frac{416}{10^5} = 0.00416$

(ii) $\frac{17}{8} = \frac{17}{2^3} = \frac{17\times 5^3}{2^3\times 5^3} = \frac{17\times 5^3}{10^3} = \frac{2125}{10^3} = 2.215$

(iv) $\frac{15}{1600} = \frac{15}{2^6\times 5^2} = \frac{15\times 5^4}{2^6\times 5^2\times 5^4} = \frac{15\times 5^4}{10^6} = \frac{9375}{10^6} = 0.009375$

(vi) $\frac{23}{2^3\times 5^2} = \frac{23\times 5^1}{2^3\times 5^2\times 5^1} = \frac{23\times 5^1}{10^3} = \frac{115}{10^3} = 0.115$

(viii) $\frac{6}{15} = \frac{2}{5} = \frac{2\times 2}{5\times 2} = \frac{4}{10} = 0.4$

(ix) $\frac{35}{50} = \frac{7}{10} = 0.7$

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If, they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120112001200012000 0...

(iii) $43.\overline{123456789}$

Ans. (i) 43.123456789

It is rational because decimal expansion is terminating. Therefore, it can be expressed in $\frac{p}{q}$ form where q = 10^9 and factors of *q* are of the form $2^n \times 5^m$ where *n* and *m* are non-negative integers

(ii) 0.120112001200012000 0...

It is irrational because decimal expansion is neither terminating nor non-terminating repeating.

(iii) $43.\overline{123456789}$

It is rational because decimal expansion is non-terminating repeating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of q **are not** of the form $2^n \times 5^m$ where n and m are non-negative integers.

Thus, $\mathbf{43.123456789} = \frac{p}{q}$, where q = 999999999