Exercise-1.3

1. Prove that $\sqrt{5}$ is irrational.

Ans. Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ **... (1)**

It means that 5 is factor of a^2

Hence, **5 is also factor of *a*** by Theorem. ... **(2)**

If, **5 is factor of *a***, it means that we can write ***a*** = 5***c*** for some integer ***c***.

Substituting value of ***a*** in **(1)**,

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

It means that 5 is factor of b^2.

Hence, **5 is also factor of *b*** by Theorem. ... **(3)**

From **(2)** and **(3)**, we can say that 5 is factor of both ***a* and *b***.

But, ***a* and *b*** are **co-prime**.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

2. Prove that ($3 + 2\sqrt{5}$) is irrational.

Ans. We will prove this by contradiction.

Let us suppose that ($3+2\sqrt{5}$) is rational.

It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that

$$\frac{a}{b} = 3 + 2\sqrt{5} \Rightarrow \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \ldots (1)$$

a* and *b are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational. It is not possible. Therefore, our supposition is wrong. ($3+2\sqrt{5}$) cannot be rational.

Hence, ($3+2\sqrt{5}$) is irrational.

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3. Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6+\sqrt{2}$

Ans. (i) We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{b}{a} \ldots (1)$$

R.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

Hence, it is irrational.

(ii) We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers *a* and *b* ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b} \quad \dots (1)$$

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

(iii) We will prove $6+\sqrt{2}$ irrational by contradiction.

Let us suppose that ($6+\sqrt{2}$) is rational.

It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that

$$6+\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \sqrt{2} = \frac{a-6b}{b} \quad \ldots (1)$$

a **and** ***b*** are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. $(6+\sqrt{2})$ cannot be rational.

Hence, $(6+\sqrt{2})$ is irrational.